**Enrollment No: ______________________ Exam Seat No: ______________________**

# C.U.SHAH UNIVERSITY

## Summer Examination-2018

**Subject Name : Engineering Mathematics – I**

**Subject Code : 4TE01EMT3** **Branch: B.Tech (All)**

**Semester : 1** **Date : 21/03/2018** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

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**Q-1** **Attempt the following questions:** **(14)**

**a)** nth derivative of $y=\frac{1}{x+a}$ is

(A) $\frac{(-1)^n n!}{(x+a)^{n+1}}$ (B) $\frac{(-1)^{n-1} n!}{(x+a)^{n+1}}$ (C) $\frac{(-1)^n n!}{(x+a)^{n}}$ (D) none of these

**b)** If $y=\cosh 2x$ and if $n$ is even, then $y_n$ equal to

(A) $2^n \sinh 2x$ (B) $2^n \cosh 2x$ (C) $\cosh(2nx)$ (D) none of these

**c)** If $y=\log(1+x)$ , then $x$ equal to

(A) $1+y+\frac{y^2}{2!}+\frac{y^3}{3!}$ (B) $1-\frac{y^2}{2!}+\frac{y^4}{4!}-.....$ (C) $y+\frac{y^2}{2!}+\frac{y^3}{3!}+\frac{y^4}{4!}+.....$
(D) none of these

**d)** If $y=\sin^{-1} x$ , then $x$ equal to

(A) $1-\frac{y^2}{2!}+\frac{y^4}{4!}-.....$ (B) $1+y+\frac{y^2}{2!}+\frac{y^3}{3!}+.....$ (C) $1-y+\frac{y^2}{2!}-\frac{y^3}{3!}+.....$

(D) $y-\frac{y^3}{3!}+\frac{y^5}{5!}-.....$

**e)** $\lim_{x\to\infty} x\left(a^{\frac{1}{x}}-1\right)=$ _____

(A) $\log_e a$ (B) 0 (C) 1 (D) none of these

**f)** $\lim_{x\to\infty}\frac{x^n}{e^{ax}}=$ _____

(A) – 1 (B) 0 (C) 1 (D) none of these

**g)** If $x=r\cos\theta,\ y=r\sin\theta,$ then

(A) $\frac{\partial x}{\partial r}=\frac{\partial r}{\partial x}$ (B) $\frac{\partial x}{\partial \theta}=0$ (C) $\frac{\partial x}{\partial r}=0$ (D) $\frac{\partial x}{\partial r}=\frac{1}{\partial r/\partial x}$

**h)** If $f(x,y)=0$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{\partial f/\partial x}{\partial f/\partial y}$ (B) $\frac{\partial f/\partial y}{\partial f/\partial x}$ (C) $-\frac{\partial f/\partial y}{\partial f/\partial x}$ (D) $-\frac{\partial f/\partial x}{\partial f/\partial y}$

**i)** If $f_1=\frac{vw}{u},\ f_2=\frac{wu}{v},\ f_3=\frac{uv}{w}$; then $\frac{\partial(f_1,f_2,f_3)}{\partial(u,v,w)}$ is equal to

(A) 0 (B) 1 (C) 3 (D) none of these

**j)** $\frac{\partial(u,v,w)}{\partial(x,y,z)}\cdot\frac{\partial(x,y,z)}{\partial(u,v,w)}=$ _____

(A) 2 (B) 1 (C) 0 (D) none of these

**k)** If $x_r=\cos\left(\frac{\pi}{2^r}\right)+i\sin\left(\frac{\pi}{2^r}\right)$ then $x_1x_2x_3.....$ to $\infty$ is

(A) $-3$ (B) $-2$ (C) $-1$ (D) 0

**l)** If $\left(\frac{1-i}{1+i}\right)^{100}=a+ib$, then

(A) $a=2,\ b=-1$ (B) $a=1,\ b=0$ (C) $a=0,\ b=1$ (D) $a=-1,\ b=2$

**m)** The system of equations $x+2y+3z=1,\ x-y+4z=0,\ 2x+y+7z=1$ has

(A) exactly one solution (B) only two solutions (C) no solution
(D) infinitely many solutions

**n)** The product of the eigenvalues of $\begin{bmatrix}1 & 4\\ 2 & 10\end{bmatrix}$ is

(A) 2 (B) 4 (C) 6 (D) 0

**Attempt any four questions from Q-2 to Q-8**

**Q-2** **Attempt all questions** **(14)**

**a)** If $y=\frac{1}{x^2+a^2}$ then find $y_n$. **(5)**

**b)** Expand $e^{\sin x}$ as a series of ascending power of $x$ upto $x^4$. **(5)**

**c)** If $u=\tan^{-1}\left(\frac{y}{x}\right)$ then verify that $\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 u}{\partial y\partial x}$ . **(4)**

**Q-3** **Attempt all questions** **(14)**

**a)** If $y=\sin\left(m\sin^{-1}x\right)$ then prove that **(5)**

$$\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}+\left(m^2-n^2\right)y_n=0$$

**b)** Prove that $\frac{x}{e^x-1}=1-\frac{1}{2}x+\frac{1}{12}x^2-.....$ **(5)**

**c)** Evaluate: $\lim_{x\to a}\log\left(2-\frac{x}{a}\right)\cot(x-a)$ **(4)**

**Q-4** **Attempt all questions** **(14)**

**a)** Evaluate: $\lim_{x\to 0}\frac{a}{x^2}\left[\frac{\sin kx}{\sin lx}-\frac{k}{l}\right]$ **(5)**

**b)** If $u=\frac{y^2}{x}$, $v=\frac{x^2}{y}$, evaluate $J=\left(\frac{x,y}{u,v}\right)$ and $J'=\left(\frac{u,v}{x,y}\right)$ and hence verify that $JJ'=1$ . **(5)**

**c)** Calculate approximate value of $\sqrt{9.12}$ by using Taylor's theorem. **(4)**

**Q-5** **Attempt all questions** **(14)**

**a)** If $u=\log\left(x^3+y^3-x^2y-xy^2\right)$ then prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right)^2 u=-\frac{4}{(x+y)^2}$ . **(5)**

**b)** Evaluate: $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x+d^x}{4}\right)^{\frac{1}{x}}$ **(5)**

**c)** If $y=\sin^4 x$ then find $y_n$ . **(4)**

**Q-6** **Attempt all questions** **(14)**

**a)** The power consumed in an electric resistor is given by $P=\frac{E^2}{R}$ (in watts). If $E=200$ volts and $R=8$ ohms, by how much does the power change if $E$ is decreased by 5 volts and $R$ is decreased by 0,20 ohms? **(5)**

**b)** Prove that $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n=\cos\left(\frac{n\pi}{2}-n\theta\right)+i\sin\left(\frac{n\pi}{2}-n\theta\right)$ **(5)**

**c)** Find the rank of matrix $A=\begin{bmatrix}6 & 1 & 1 & 1\\ 16 & 1 & -1 & 5\\ 7 & 2 & 3 & 0\end{bmatrix}$. **(4)**

**Q-7** **Attempt all questions** **(14)**

**a)** Using matrix method, show that the equations **(5)**
$3x+3y+2z=1,\ x+2y=4,\ 10y+3z=-2,\ 2x-3y-z=5$
Are consistent and hence find the solution.

**b)** Find the roots common to the equations $x^4+1=0$ and $x^6-i=0$ . **(5)**

**c)** Find $\tanh x$ if $5\sinh x-\cosh x=5$. **(4)**

**Q-8** **Attempt all questions** **(14)**

**a)** Find the inverse of $A=\begin{bmatrix}-1 & 1 & 2\\ 3 & -1 & 1\\ -1 & 3 & 4\end{bmatrix}$ by Gauss-Jordan reduction method. **(5)**

**b)** Using De Moivre's theorem, expand $\sin^8\theta$ in a series of cosines of multiples of $\theta$ . **(5)**

**c)** Check whether the following set of vectors is linearly dependent or linearly independent: **(4)**
$(1,2,-1,0),\ (1,3,1,2),\ (4,2,1,0),\ (6,1,0,1)$